

MATH 2B Review: Product & Quotient Rule

Facts to Know:

Rules for taking the derivative of a product or quotient of functions.

Product Rule: $\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

Examples:

1. Compute the derivative of $y = x^3 \cos(x)$.

$$\begin{aligned} f(x) &= x^3 \\ g(x) &= \cos(x) \\ f'(x) &= 3x^2 \\ g'(x) &= -\sin(x) \end{aligned} \quad \begin{aligned} \frac{dy}{dx} &= f'(x)g(x) + f(x)g'(x) \\ &= 3x^2 \cos(x) + x^3 (-\sin(x)) \\ &= 3x^2 \cos(x) - x^3 \sin(x) \end{aligned}$$

2. Use quotient rule to show the derivative of $\tan(x)$ is $\sec^2(x)$.

$$\begin{aligned} \frac{d}{dx} \tan(x) &= \frac{d}{dx} \frac{\sin(x)}{\cos(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} \\ f(x) &= \sin(x) \\ g(x) &= \cos(x) \\ f'(x) &= \cos(x) \\ g'(x) &= -\sin(x) \end{aligned} \quad \begin{aligned} &= \frac{\cos(x) \cdot \cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} \\ &= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} \\ &= \frac{1}{\cos^2(x)} = \sec^2(x) \end{aligned}$$